Seamless Data-Rate Change Using Punctured Convolutional Codes for Time-Varying Signal-to-Noise Ratio

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Abstract

in a time-varying signal-to-noise ratio (t$'Nil) environment, symbol rate is often changed to maximize data return. However, the symbol-rate change has some undesirable effects such as changing the transmission bandwidth and perhaps causing the receiver symbol loop to lose lock temporarily, thus losing some data. In this article, we are proposing an alternate way of varying the data rate without changing the symbol rate and therefore the transmission bandwidth. The data rate change is achieved in a seamless fashion by puncturing the convolutionally encoded symbol stream to adapt to the changing SNR environment. We have also derived an exact expression to enumerate the number of unique puncturing patterns. To demonstrate this seamless rate-change capability, we searched for good puncturing patterns for the Galileo (14,1/4) convolutional code and changed the data rates by using the punctured codes to match the Galileo SNR profile of November 9, 1997. We show that this scheme reduces the symbol-rate changes from 9 to 2 and provides a larger data return in a day and a higher symbol SNR during most of the day.

1 Introduction

In deep-space communications and other space communications, the signal-to-noise ratio (SNR) varies during a day. The degree of variation is determined by weather conditions, antenna elevation angle, antenna pointing accuracy (both the transmitter and receiver antennas), changes in satellite latitude, and many other factors. For example, the total signal-power-to-noise-density ratio (P_t/N_o) during a typical 24-hour pass of the

Galileo Mission can fluctuate in a range between 16 and 22 dB-Hz. In order to maximize the data return in this time-varying SNR environment, the transmitted symbol rate is varied as a function of the estimated P_t/N_0 at the antenna. The symbol rate is set as high as possible under the constraint that the symbol SNR is high enough for the tracking loops to remain in lock and that the bit-error-rate (BER) requirement is met. In the Galileo Mission, there arc six different symbol rates, and there can be as many as eight symbol rate changes (from 10 to 640 symbols/second) during a day. One problem associated with the symbol-rate change at low operating symbol SNR is that the symbol synchronization loop may have to reacquire the symbol phase, which may cause real-time data loss. A technique that involves opening the symbol loop at the moment of the sy~nbol-rate change has been proposed [1], but this technique requires very accurate time predicts on the moment of change. It is not clear if the predict information can be obtained within the required accuracy.

In this article, we arc proposing a simple and low-cost alternative solution to the data-rate change problem by changing the data rate at the error-correcting coding stage rather than at the transmission stage. The data rate is changed by puncturing the low rate convolutional code while the symbol rate is kept constant. In this way, the basic structures of the encoder and decoder remain unchanged making the scheme simple and less costly. The idea is to minimize the number of symbol rate changes and still maintain a nigh enough symbol SNR for the loops to remain in lock and the BER to stay low. Symbol rate is changed only if the symbol SNR goes too high (wasting bit SNR) or too low (making the receiver unable to track the symbols).

By allowing the code rate change, we arc essentially adding a degree of freedom in the data-return maximization problem. 'The code rate will share a part of the necessary data-rate changes with the symbol rate, therefore reducing the number of symbol rate changes. This feature becomes even more important when the available bandwidth is fixed.

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[1] J. Berner, "GLL Data Rate Changes", Notes, June 11, 1993. (Jet]'repulsion laboratory, Pasadena, CA, Internal Document)

in Section 2, we will present the definition and an overview of puncturing patterns. in Section 3, wc will discuss our procedure for selecting good puncturing patterns. in Section 4, wc will provide an example of using the punctured convolutional code for the SNR profile of the Galileo Mission on November 9, 1997, which is an arbitrarily chosen day. in Section 5, wc will give some concluding, remarks.

2 Definition and Enumeration of Puncturing Patterns

2.1 Definition of Puncturing Patterns

A regular rate $1/N$ convolutional code generates N code symbols per bit. By periodically and systematically refraining from transmission of some of the code symbols, a higher rate code can be constructed from an original lower rate $1/N$ code. Let the period be L bits or NL code symbols. Wc define a puncturing pattern 1' of period NL symbols to be an NL binary tuple where a 1 denotes that the symbol in the corresponding location is to be sent, and a O denotes that the symbol is to be deleted. If there arc m zeros in P, the resulting punctured code is a higher rate $L/(NL-m)$ code, where $O \leq m < (N-1)L$. For example, let $N = 4$ and $L = 4$, and onc puncturing pattern is $P = \{0111\ 11101011\ 1101\}$. We define the rightmost digit to correspond to the first symbol and the rightmost group of four digits to correspond to the four symbols of the first bit. The puncturing pattern, P, indicates that the second symbol in the first bit, the third symbol in the second bit, the first symbol in the third bit, and the fourth symbol in the fourth bit in a period arc not transmitted. The resulting punctured code is a rate $4/(4 \times 4 - 4) = 1/3$ code. With the leftmost digit being the most significant bit and the rightmost digit being the least significant bit, the puncturing pattern P can be represented as $dbe7$ in hexadecimal form.

2.2 Enumeration of Puncturing Patterns

in this section, wc develop an exact expression to enumerate the number of unique puncturing patterns.

Clearly, there are $\binom{NL}{m}$ different possible patterns for P. Since P is periodic in NL symbols, any cyclic shift of N symbols in P gives the same code performance as P. However, this does not reduce the number of patterns that give a distinct code performance by a factor of N, as some of the $\binom{NL}{m}$ patterns may have a smaller period L_i. That is, L_i divides L, which is denoted by $L_i \mid L$. Let $f(L_i)$ denote the number of puncturing patterns with period L_i exactly (including 1). Notice that $f(L_i) = 0$ if the m zeros cannot be evenly divided among $\frac{L}{L_i}$ partitions (i. e., $\frac{L}{L_i} \not\mid m$). Also among the $\binom{NL_i}{\frac{mL_i}{L}}$ patterns with period L_i, some may have smaller periods. Let p be a prime that divides L_i (not including L). If $p \mid \frac{mL_i}{L}$, then there are $\binom{\frac{NL_i}{p}}{\frac{mL_i}{Lp}}$ patterns of P with period $\frac{L_i}{p}$. The total number of distinct puncturing patterns is therefore

$$\sum_{L_i \mid L} \frac{1}{L_i} f(L_i),$$

where $f(L_i)$ can be enumerated as follows:

$$f(L_i) = \binom{NL_i}{\frac{mL_i}{L}} \cdot \sum_{p \mid L_i} \binom{\frac{NL_i}{p}}{\frac{mL_i}{Lp}}.$$

Notice that we define the combinatorics function $\binom{m}{n} = 0$ if either m or n is not an integer. In the above example with $N = 4$, $L = 4$, and $m = 4$, an exhaustive search requires checking $\binom{16}{4} = 1820$ puncturing patterns. By taking into account the cyclic property of the puncturing patterns, the number of unique puncturing patterns is now reduced to

$$\frac{1}{4}\left[\binom{16}{4} - \binom{8}{2}\right] + \frac{1}{2}\left[\binom{8}{2} - \binom{4}{1}\right] + \binom{4}{1} = 464,$$

which is a reduction by almost a factor of 4.

3 Puncturing Pattern Search Procedure

in this section we describe the search procedure that we used to find good puncturing patterns for a rate 1/N convolutional code. Using this procedure, we searched for punctured codes for a low-rate convolutional code, the (14,1 /4) code used for Galileo.

We punctured it from rate 1/4 to rate 1/3, then to rate 1 /2. A rate compatibility [1] restriction is added in the puncturing pattern search. That is, a code bit used in the hig]l-rate code is also used in the]ow-rate code. For example, to search for a rate-1 /2 punctured code, we puncture the rate-1/3 code found a step before, not the rate-1/4 code. This was necessary mainly because of limited computing resources.

For each punctured code rate, the goal is to find the puncturing pattern P that gives the lowest BER at that rate for a range of SN R values. Direct simulation of the punctured convolutional code is not viable since there al c so many different puncturing patterns. As a first step to select the puncturing patterl **Is, wc used** the resulting weight profile of the punctured code that includes the maximum free distance, d_{free}, the number of paths of weight d, a_d, and the information bit error weight c_d as criteria. To further simplify our search we only searched for paths of weight d such that $d_{free} \leq d \leq d_{cut}$, where d_{cut} is some pre-determined value that is large enough to infer the code's BER performance and small enough to complete the search in a reasonable time. Note that there are L different starting points for diverging paths, where L is the period of the puncturing patterns. The worst case is considered in comparing the puncturing patterns.

A systematic search is carried out to find the patterns with the maximum free distance and the minimum number of paths of weight d for Viterbi decoding. Three patterns with the largest free distance and smallest number of paths of weight d are then simulated to obtain a BER curve. The resulting BERs are used further to compute the BERs for the concatenated codes of convolutional code as the inner code and the Reed-Solomon (RS) (255,223) code as the outer code assuming infinite interleaving.

Once we have the points of E_b/N_o versus BER of the concatenated code, we fit a curve through these points. These curves are used to determine the BER for a given SNR profile of the Galileo Mission. When the BER is less than 10-7, we determine that the code rate can be used in that time period.

Note that the (14,1/4) Galileo code is used here only to demonstrate the alternative possibility of using punctured codes, In fact, the (14,1 /4) code is composed of a (1 1,1 /2)
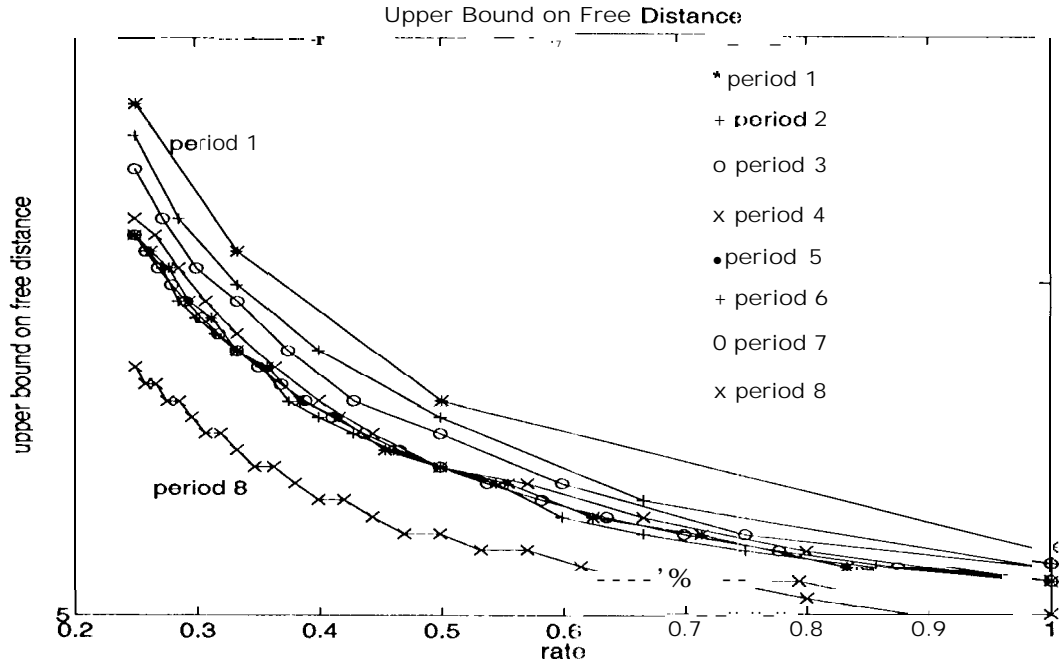
Upper Bound on Free Distance

Figure 1: Upper bounds on free distance for punctured codes from (14,1 /4) code.

convolutional code and the NASA standard (7,1 /2) code. The NASA standard (7,1 /2) code was necessary because the hardware encoder on the spacecraft cannot be altered or bypassed.

3.1 Upper Bound on Free Distance

Before searching for the maximum free distances, we compute the upper bounds of the free distances to see the effect of the puncturing period on the free-distance bound of the punctured codes. The upper bounds on the free distances for convolutional codes can be computed using expressions given in [2]. Fig. 1 shows some of the bounds on the free distances for codes punctured from code (1 4,1/4) with the minimum period from 1 to 8. By minimum period we mean that the period 4 dots not include period 1 or 2. The results show that a shorter puncturing period gives a higher upper bound on the free distance, but the shorter puncturing period provides a smaller set of possible code rates.

3.2 **Weight Spectra of Punctured Codes**

The parent code in this case has the following polynomials: 2c22, $3d7d$, $2bcd$, and $1dd3$. First, we search for punctured codes from (14,1 /4) to (14,1 /3) and find the weight spectra corresponding to all different punctured patterns. The period in this case is 4, which corresponds to 464 different puncturing patterns. We then sort the weight spectra in the ascending order according the number of paths of weight d, a_d. Finally, we pick the best three patterns and their weight spectra are shown in 'J'able 1. According to the weight spectra, the best pattern is $bbbb$. This implies that the puncturing pattern has period 1, and the third symbol is punctured out every time. This corresponds to the (14,1 /3) code with polynomials: 2c22, $3d7d$, and $1dd3$.

To further puncture the code to rate 1 /2, we use the best 1/3 code found earlier as the parent code. The following patterns are found to be the best: 3636, 3535, "and 3333 in octal numbers. The weight spectra of the three best puncturing patterns are shown in '1'able 2.

Note that when searching for puncturing patterns with period 4, those patterns with period 1 and *2* are included.

Table 1

Weight Spectra of Punctured Codes (14,1 /3) From (14,1 /4) Mother Code														
pattern	d	23	24	25	26	27	28	29	30	31	32	33	34	35
bbbb	a_d	0	4	6	4	9	14	22	48	83	130	237	389	638
bdbd	a_d	1	1	3	8	13	21	27	54	68	137	225	**400**	**652**
bbbd	a_d	1	5	5	5	13	16	38	54	101	146	288	481	800
bbbb	c_d	0	14	18	88	55	72	122	332	641	920	1853	3134	5530
bdbd	c_d	1	*3*	9	35	60	121	139	203	486	*938*	*1699*	**3150**	**5368**
bbbd	c_d	5	*14*	19	224	77	919	240	403	724	1080	2313	4067	7068

Table 2

pattern	d	13	14	15	16	17	18	19	20	21	22	23	24
				Weight Spectra of Punctured Codes (1 4,1 /2) From (1 4,1 /3) Mother Code									
3636	a_d	0	2	6	10	24	51	142	344	824	1956	4726	11363
3535	a_d	0	2	8	9	35	70	154	371	931	2286	5464	113234
3333	a_d	0	3	0	14	0	73	0	545	0	2884	0	116679
3636	c_d	0	5	20	70	1466	3354	11444	29144	7780	20229	52967	5525
3535	c_d	0	9	37	53	251	550	1298	3370	9353	25245	64261	35749
3333	c_d	0	9	0	71	0	520	0	4686	0	29943	0	4011

3.3 BER of Punctured Convolutional Code from Simulation

The weight,-spectra search is only the first step in the code puncturing pattern search. To further compare their performance, the punctured codes are simulated with an encoder and the Viterbi decoder for several bit- SNR values. The traceback length used in the Viterbi decoder in this case is at least 160 and the input soft symbols are quantized with 8 bits. The simulated results are shown in Tables 3 and 4. Generally, the three puncturing patterns give similar BERs.

Table 3

E_b/N_o	BER with punctured patterns		
	bbbb	*bdbd*	*bbbd*
-1.2494	0.3528	0.3532	0.3533 "_
-0.7494	0.2320	0.2336	0.2330
-0.2494	0.1083	0.1100	0.1090
0.2506	0.0342	0.0345	0.0347
0.7506	0.0076	0.0077	0.0076
1.2506	0.0012	0.0012	0.0012

BER of Punctured Convolutional Codes Using Punctured Codes (14,1/3)

'l'able 4

E_b/N_o	BER with punctured patterns		
	3636	3535	3333
-1.0103	0.4389	0.4339	0.4445
-0.5103	0.3573	0.3519	0.3625
-0.0103	0.2230	0.2206	0.2279-
0.4897	0.0924	0.0934	0.0924
0.9897	0.0230	0.0243	0.0226
1.4897	0.0040	0.0043	0.0035
1.9897	0.0005	0.0005	0.0004
2.4897	4.7e-05	5.4e 05	3.5e-05

BER of Punctured Convolutional Codes Using Punctured Codes (14,1 /2)

3.4 BER of Concatenated Code

Once we obtain the BER from the Viterbi decoder, we can compute the bit-error rate at the output of the Reed-Solomon decoder assuming infinite interleaving using expression given in [3,p256]. In the case of Galileo Mission, there are 8 bits in a code word 255 codewords in a frame, and the number of correctable errors is 16. The computed BER at the output of the RS decoder s shown in Tables 5 to 7.

Table 5

BER Output to RS Decoder
Using Punctured Code (1 4,1 /4)

E_b/N_o	BER "input to RS decoder	BER output of RS decoder
-2.0	0.4218	0.4218
-1.5	0.3408	0.3408
-1.0	0.2139	0.2139
-0.5	0.1023	0.1023
0.0	0.0326	0.0194
0.5	0.0070	5.4e-9
1.0	'0.0013	2.3e-20
1.5	0.0002	1.1e-49

Table 6

E_b/N_o	BER input to RS decoder	BER output of RS decoder
BER Output of RS Decoder Using Punctured Codes (14,1/3)		
-1.2494	0.3528	0.3528
-0.7494	0.2320	0.2320
-0.2494	0.1083	0.1083
0.2506	0.0342	0.0230
0.7506	0.0076	1.8e-8
1.2506	0.0012	1.0e-20

Table 7

E_b/N_o	BER input to RS decoder	BER output of RS decoder
BER Output of RS Decoder Using Punctured Code (14,1/2)		
-1.0103	0.4389	0.4389
-0.5103	0.3573	0.3573
-0.0103	0.2230	0.2230
0.4897	0.0924	0.0924
0.9897	0.0230	0.0029
1.4897	0.0040	1.6e-12
1.9897	0.0005	3.7e-27
2.4897	4.7e-05	1.6e-44

4 Example Using Galileo Profile

We use the predicted SNR profile of the Galileo mission on November 9, 1997 as an example to explain how the number of symbol-rate changes can be reduced with code rate changes. For a given SNR profile, for example, the one shown in Fig. 2, the objective is to get the maximum data return under the conditions that the bit-error rate is below 10^{-7}, and that the symbol SNR is maintained above -6 dB for the carrier, subcarrier, and symbol loops to track. To achieve this goal, the current plan is to change the symbol rate using a fixed code rate, 1/4, and an alternate way is to allow code rate to change as WC]], thus reducing the number of symbol-rate changes.
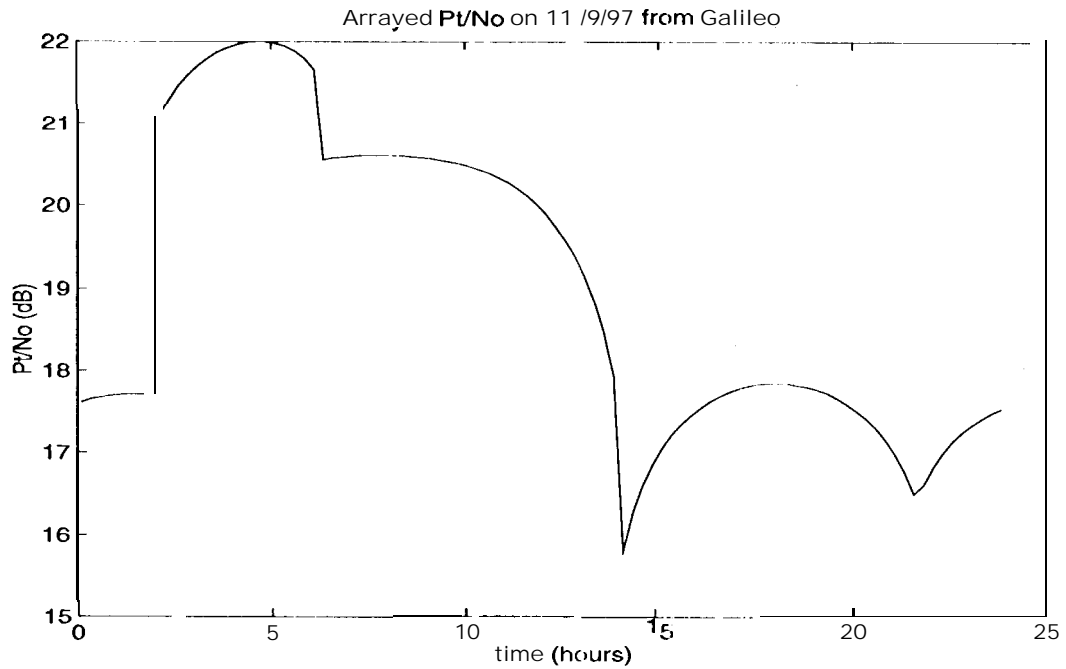
Figure 2: Arrayed P_t/N_o on 11 /9/97 from Galileo.

We arbitrarily select a set of three code rates, namely 1 /4, 1 /3, and 1 /2. The variable code rate can on] y take values from this set,. Fig. 3 shows the symbol rates using fixed and variable code rates. in the fixed-code-rat e case, there arc 9 symbol-rate changes, compared to 2 symbol-rate changes in the variable code rate case. With these symbol rates, each of the two systems will have the symbol SNR above -6 dB as required, where the variable-code-rate case has a slightly higher symbol SNR for most of the day. The code rate changes arc show in Fig. 5.

Multiplying the code rates by the symbol rates, wc obtain the bit rates as shown in Fig. 6 for the fixed and variable code-rate cases. The total data returns in the day arc the areas under the two curves in Fig. 6. The data return using the variable code-rate is 4.5075 M-bits, which is slightly higher than the 4.2921 M-bits using the fixed code rate.

5 Conclusions

in this paper wc have described a simple and low-cost method to change the data rate
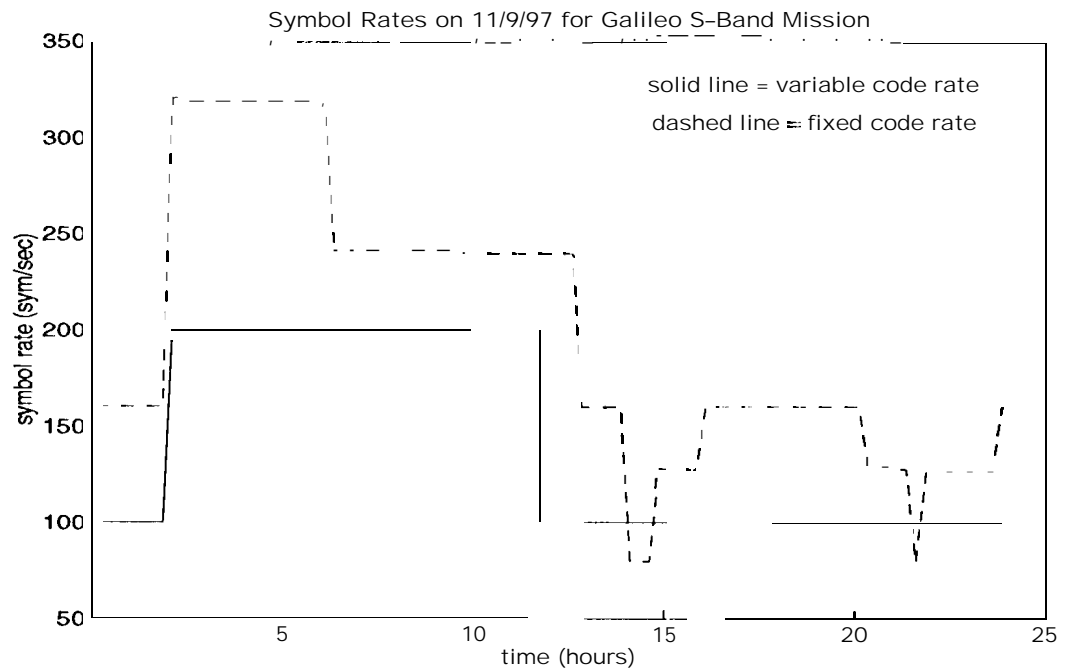
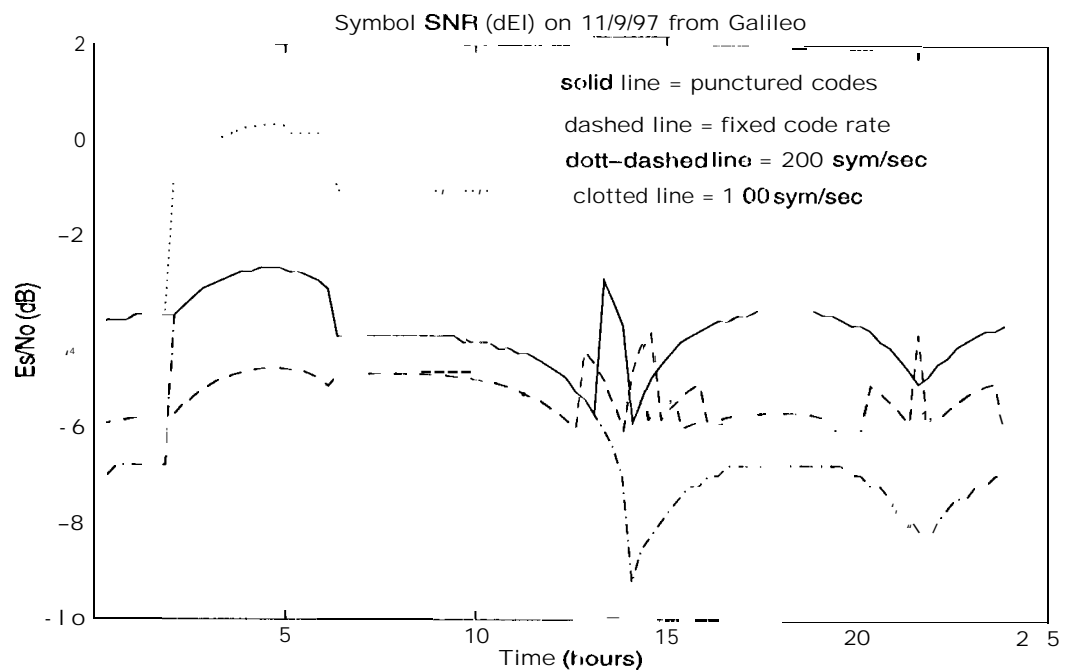Figure 3: Symbol Rates on 11 /9/97 for Galileo using fixed and variable code rates.

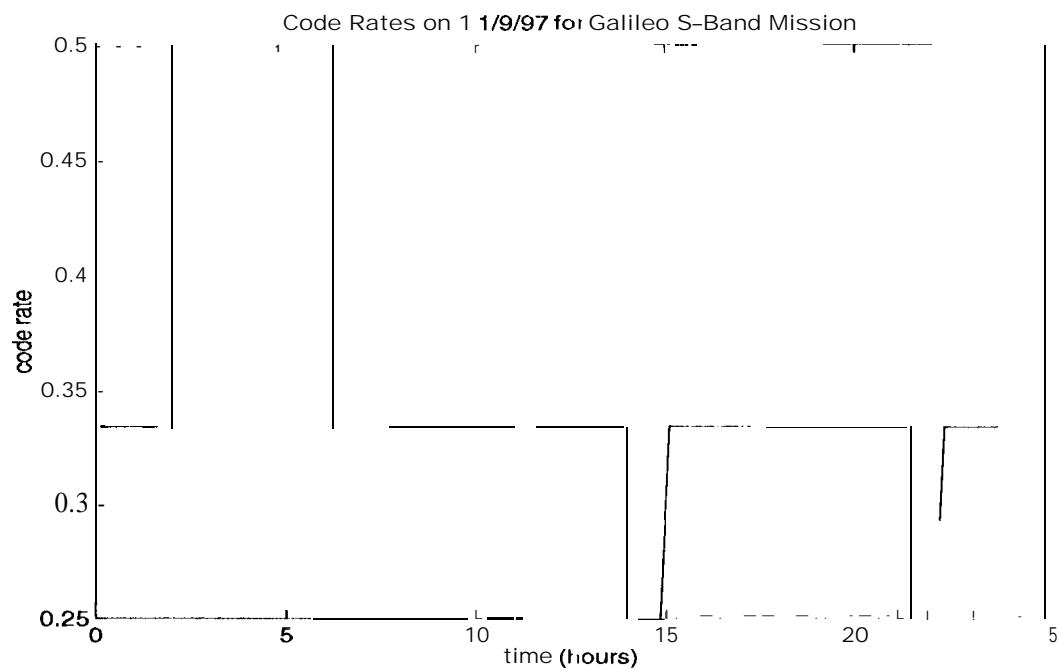Figure 4: Symbol SNR on 11 /9/97 for Galileo using fixed and variable **code** rates.
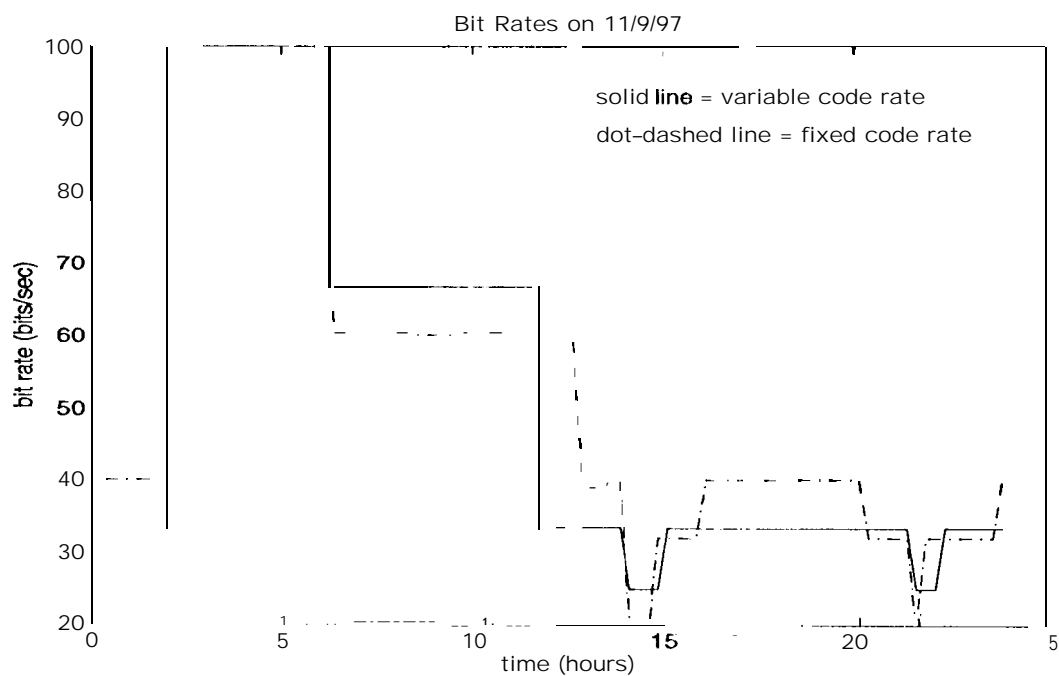
Code Rates on 1 1/9/97 foɪ Galileo S-Band Mission

Figure 5: Variable Code rates on 11/9/97 for Galileo Mission.

Bit Rates on 11/9/97

solid line = variable code rate

dot-dashed line = fixed code rate

Figure 6: Bit rates on 11 /9/97 for Galileo.

to match the time-varying P_t/N_0 environment. This is done by puncturing the convolutional code at the error-correction coding stage rather than by changing the symbol rate at the transmission stage. The main advantages of this method are that it allows seamless transition from one data rate to another and that for a fixed available bandwidth data rate is allowed to change for a larger data return. We applied this method to the Galileo SNR profile on November 9, 1997 as an example to demonstrate its effectiveness. We showed how this method reduces the number of symbol rate changes from 9 to 2, and gives a slightly larger data return in a day and higher symbol SNR for most of the day.

Notice that in this example, we arbitrarily picked 100 and 200 symbols/sec as two symbol rates to be used. We are developing techniques to select the symbol rates that will maximize the data return. The problem is formulated below.

For a given SNR profile $\frac{P_t}{N_o}(t)$, we wish to find the symbol rate $R_{sym}(t)$ and the code rate $R_c(t)$ such that the data return given by $\int_{t_1}^{t_i} R_{sym}(t) R_c(t) dt$ is maximum subject to the following constraints:

1. the number of changes on symbol rate R_{sym} is less than a desired number,

2. the BER is below a designed value, $BER < BER_{design}$,

3. the symbol SNR is above the minimum value for the tracking loops to maintain lock, $E_s/N_o > (E_s/N_o)_{min}$.

Acknowledgment

References

1. J. Hagenauer, "Rate-Compatible Punctured Convolutional Codes (RCPC Codes) and 'J'heir Applications", *IEEE Trans. on Comm.*, Vol. 36, No. 4, pp. 389-400, April 1988.

2. D. G. Daut, J. W. Modestino, and 1,. 1). Wismer, "New Short Constraint Length Convolutional Code Constructions for Selected Rational Rates", *IEEE Trans. on Information Theory*, Vol. IT-28, No. 5, pp. 794-800, September 1982.

3. J. 11. Yuen, *Deep Space Telecommunications Systems Engineering*, New York: Plenum Press, 1983.